

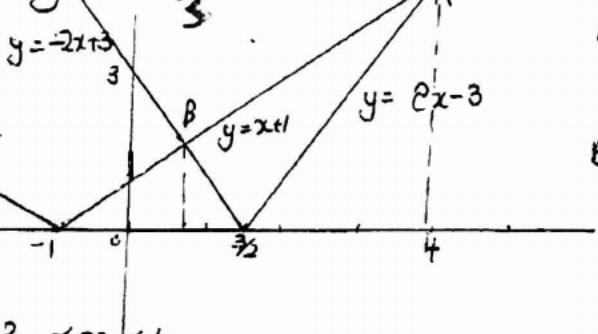
Core 3 June 2006

$$1) y = \sqrt{4x+1} = (4x+1)^{\frac{1}{2}}$$

$$(1, 3) x=2 \quad \frac{dy}{dx} = \frac{2}{3}$$

$$\frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}} \times 4 = \frac{2}{4x+1}$$

$$y - 3 = \frac{2}{3}(x-2)$$



$$A \quad 2x-3 = 2+1$$

$$x = 4$$

$$B \quad -2x+3 = x+1$$

$$2 = 3x$$

$$x = \frac{2}{3}$$

$$2) |2x-3| < |x+1| \quad y = -x-1$$

$$|2x-3| < |x+1| \text{ when } \frac{2}{3} < x < 4$$

$$3) f(x) = 2x^3 + 4x - 35. \quad f(-2) = 16 + 8 - 35 = -11 \quad f(3) = 54 + 12 - 35 = +21$$

change of sign -ve to +ve between  $x = 2$  and  $3$ .

$$x_{n+1} = \sqrt[3]{17.5 - 2x_n} \quad x_1 = 2 \quad x_2 = \sqrt[3]{17.5 - 4} = 2.3811 \quad x_3 = \sqrt[3]{17.5 - 2 \times 2.3811} = 2.3354$$

$$\text{CALC. & ENTER } (17.5 - (2 \text{ANS}))^{1/3} \text{ ENTER } x_4 = 2.3410 \quad x_5 = 2.3403 \quad x_6 = 2.3404 \quad x_7 = 2.3404$$

root is 2.34 (2dp).

$$4) y = 5^{x-1} \quad \ln y = \ln 5^{x-1} \quad \ln y = (x-1)\ln 5 \quad \ln y + \ln 5 = x\ln 5$$

$$x = \frac{\ln y}{\ln 5} + 1 \quad \frac{dx}{dy} = \frac{1}{\ln 5} \times \frac{1}{y} \quad \text{gradient of curve } \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = y \ln 5$$

$$\frac{dy}{dx} = \frac{5^{x-1}}{5^{x-1} \ln 5}. \quad \text{at } (3, 25) \quad \frac{dy}{dx} = \frac{25}{25 \ln 5} = \frac{1}{\ln 5}.$$

$$5) i) \sin 2\theta = 2\sin\theta \cos\theta \quad \sin\theta = \frac{1}{4} \quad \begin{array}{c} \triangle \\ \alpha \\ 4 \\ \sqrt{16-16} \end{array} \quad \cos\alpha = \frac{\sqrt{15}}{4}$$

$$\sin 2\theta = 2 \times \frac{1}{4} \times \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{8}$$

$$ii) 5 \times 2\sin\beta \cos\beta \frac{1}{\cos\beta} = 3 \quad 10\sin\beta \cos\beta - 3\cos\beta = 0 \quad \cos\beta(10\sin\beta - 3) = 0$$

$$0 < \beta < 90 \quad \cos\beta = 0 \text{ not required solution} \quad \sin\beta = 0.3 \quad \beta = 17.5^\circ (3sf).$$

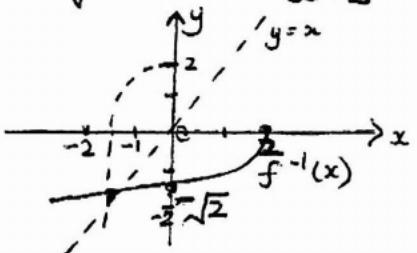
$$6) i) f(x) = 2 - x^2 \quad x \leq 0 \quad f(3) = 2 - (-3)^2 = -7 \quad f(-7) = 2 - (-7)^2 = -47$$

$$ff(-3) = -47.$$

$$ii) x \rightarrow \frac{x^2}{-x^2} \rightarrow \text{change sign} \rightarrow \frac{-x^2}{x^2} \rightarrow -x^2 + 2$$

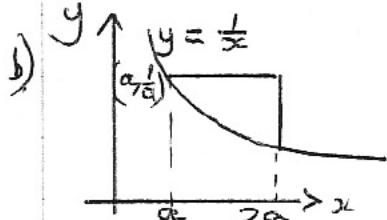
$$x \rightarrow \frac{x-2}{2-x} \rightarrow \text{change sign} \rightarrow \frac{2-x}{x-2} \rightarrow \pm \sqrt{2-x} \quad \text{domain } f(x) \quad x \leq 0$$

$$f^{-1}(x) = \sqrt{2-x} \quad x \leq 2 \quad (\text{negative square root as reflection of } f(x) \text{ in } y=x)$$



graph meets axis  $(2, 0), (0, \sqrt{2})$

$$7) \int_1^2 \frac{2}{(4x-1)^2} dx = \int_1^2 2(4x-1)^{-2} dx = \left[ \frac{2(4x-1)^{-1}}{-1 \times 4} \right]_1^2 = \left[ \frac{1}{2(4x-1)} \right]_1^2 \\ = \left( -\frac{1}{2 \times 7} \right) - \left( -\frac{1}{2 \times 3} \right) = -\frac{1}{14} + \frac{1}{6} = -\frac{6+14}{84} = \frac{8}{84} = \frac{2}{21}$$



$$\text{Area rectangle} = a \times \frac{1}{a} = 1$$

Area under  $y = \frac{1}{x}$  from  $x=a$  to  $x=2a$ .

$$\int_a^{2a} \frac{1}{x} dx = [\ln x]_a^{2a} = \ln 2a - \ln a = \ln 2.$$

$$\text{Shaded area} = 1 - \ln 2 = \ln e^1 - \ln 2 = \ln \left(\frac{e}{2}\right)$$

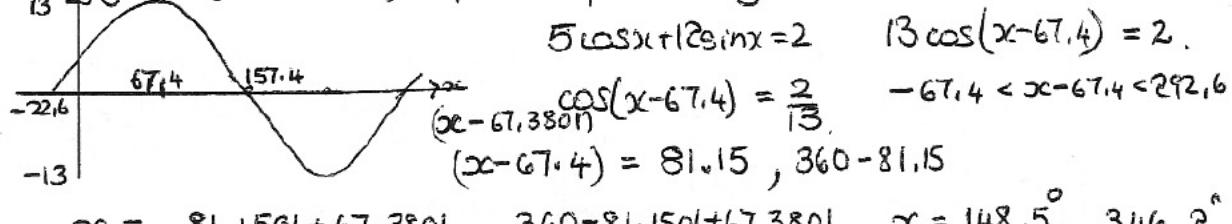
$$8. 5\cos x + 12\sin x \equiv R\cos(x-\alpha) \quad R\cos(x-\alpha) = R\cos x \cos \alpha + R\sin x \sin \alpha$$

$$\frac{R\sin \alpha}{R\cos \alpha} = \frac{12}{5} \quad \tan \alpha = \frac{12}{5} \quad \alpha = 67.38^\circ. \quad R^2 \sin^2 \alpha + R^2 \cos^2 \alpha = 144 + 25 \\ R^2 (\cos^2 \alpha + \sin^2 \alpha) = 169 \quad \cos^2 \alpha + \sin^2 \alpha = 1 \quad R = 13$$

$$5\cos x + 12\sin x = 13\cos(x - 67.4^\circ)$$

Translate  $y = \cos x$  by  $67.4^\circ$  in a positive direction parallel to  $x$ -axis

Stretch  $y = \cos(x-67.4^\circ)$  by a factor 13 parallel to  $y$ -axis



$$5\cos x + 12\sin x = 2 \quad 13\cos(x - 67.4^\circ) = 2.$$

$$\cos(x - 67.4^\circ) = \frac{2}{13}. \quad -67.4 < x - 67.4 < 292.6$$

$$(x - 67.4) = 81.15, 360 - 81.15$$

$$x = 81.15 + 67.3801, 360 - 81.15 + 67.3801 \quad x = 148.5^\circ, 346.2^\circ$$

$$9. V = \pi \int x^2 dy. \quad y = 2\ln(x-1) \quad \frac{dy}{dx} = \ln(x-1) \quad e^{\frac{y}{2}} = x-1 \quad x = e^{\frac{y}{2}} + 1$$

$$V = \pi \int_0^P (e^{\frac{y}{2}} + 1)^2 dy = \pi \int_0^P e^y + 2e^{\frac{y}{2}} + 1 dy$$

$$V = \pi \left[ e^y + 4e^{\frac{y}{2}} + y \right]_0^P = \pi \left( e^P + 4e^{\frac{P}{2}} + P \right) - \pi (1 + 4 + 0)$$

$$V = \pi \left( e^P + 4e^{\frac{P}{2}} + P - 5 \right).$$

$$\text{In general } p=y \quad V = \pi \left( e^y + 4e^{\frac{y}{2}} + y - 5 \right) \quad \frac{dV}{dy} = \pi \left( e^y + 2e^{\frac{y}{2}} + 1 \right)$$

$$\text{given } \frac{dy}{dt} = 0.2 \text{ cm min}^{-1} \quad \frac{dV}{dt} = \frac{dV}{dy} \times \frac{dy}{dt}$$

$$\frac{dV}{dt} = \pi \left( e^y + 2e^{\frac{y}{2}} + 1 \right) \times 0.2 \quad \text{when } y = p = 4$$

$$\frac{dV}{dt} = 0.2\pi \left( e^4 + 2e^2 + 1 \right) = 44 \text{ cm}^3 \text{ min}^{-1} \quad (2 \text{ sf})$$